

Instructions: Complete each of the following exercises for practice.

1. Use the chain rule to compute $\frac{df}{dt}$.
 - (a) $f(x, y) = xy^3 - x^2y$; $x(t) = t^2 + 1$, $y(t) = t^2 - 1$
 - (b) $f(x, y) = \frac{x-y}{x+2y}$; $x(t) = e^{\pi t}$, $y(t) = e^{-\pi t}$
 - (c) $f(x, y) = \sin(x) \cos(y)$; $x(t) = \sqrt{t}$, $y(t) = \frac{1}{t}$
 - (d) $f(x, y) = \sqrt{1+xy}$; $x(t) = \tan(t)$, $y(t) = \arctan(t)$
 - (e) $f(x, y, z) = x \exp(\frac{y}{z})$; $x(t) = t^2$, $y(t) = 1 - t$, $z(t) = 1 + 2t$
 - (f) $f(x, y, z) = \ln(\sqrt{x^2 + y^2 + z^2})$; $x(t) = \sin(t)$, $y(t) = \cos(t)$, $z(t) = \tan(t)$
2. Use the chain rule to compute $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.
 - (a) $z(x, y) = (x - y)^5$; $x(s, t) = s^2t$, $y(s, t) = st^2$
 - (b) $z(x, y) = \arctan(x^2 + y^2)$; $x(s, t) = s \ln(t)$, $y(s, t) = te^s$
 - (c) $z(x, y) = \ln(3x + 2y)$; $x(s, t) = s \sin(t)$, $y(s, t) = t \cos(s)$
 - (d) $z(x, y) = \sqrt{x}e^{xy}$; $x(s, t) = 1 + st$, $y(s, t) = s^2 - t^2$
 - (e) $z(r, \theta) = e^r \cos(\theta)$; $r(s, t) = st$, $\theta(s, t) = \sqrt{s^2 + t^2}$
 - (f) $z(u, v) = \tan(\frac{u}{v})$; $u(s, t) = 2s + 3t$, $v(s, t) = 3s - 2t$
3. A function $f(x, y)$ with $\text{dom}(f) = \mathbb{R}^2$ is *n-homogeneous* when $f(tx, ty) = t^n f(x, y)$ for all t (where $n > 0$).
 - (a) Verify that $g(x, y) = x^2y + 2xy^2 + 5y^3$ is 3-homogeneous.
 - (b) Assume f is *n-homogeneous* and has continuous second-order partial derivatives. Prove the following.
 - i. $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = n f(x, y)$
 - ii. $x^2 \frac{\partial^2 f}{\partial x^2} + 2xy \frac{\partial^2 f}{\partial x \partial y} + y^2 \frac{\partial^2 f}{\partial y^2} = n(n-1) f(x, y)$
 - iii. $\frac{\partial}{\partial x} [f(tx, ty)] = t^{n-1} f_x(x, y)$
4. Compute the gradient of the function.

(a) $f(x, y) = \frac{x}{y}$ (b) $f(x, y) = x^2 \ln(y)$	(c) $f(x, y, z) = x^2yz - xyz^3$ (d) $f(x, y, z) = y^2e^{xyz}$
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5. Compute the directional derivative $D_{\mathbf{u}}f$ at point P in the direction of angle θ .
 - (a) $f(x, y) = xy^3 - x^2$; $P = (1, 2)$, $\theta = \frac{\pi}{3}$
 - (b) $f(x, y) = y \cos(xy)$; $P = (0, 1)$, $\theta = \frac{\pi}{4}$
 - (c) $f(x, y) = \sqrt{2x + 3y}$; $P = (3, 1)$, $\theta = -\frac{\pi}{6}$
6. Find the directional derivative of f at the point P in the direction of $\mathbf{v}$.
 - (a) $f(x, y) = \frac{x}{x^2 + y^2}$; $P = (1, 2)$, $\mathbf{v} = \langle 3, 5 \rangle$
 - (b) $f(u, v) = u^2e^{-v}$; $P = (3, 0)$, $\mathbf{v} = \langle 3, 4 \rangle$
 - (c) $f(x, y, z) = x^2y + y^2z$; $P = (1, 2, 3)$, $\mathbf{v} = \langle 2, -1, 2 \rangle$
 - (d) $f(r, s, t) = \ln(3r + 6s + 9t)$; $P = (1, 1, 1)$, $\mathbf{v} = \langle 4, 12, 6 \rangle$